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III

Total No. of Questions – 24
Total No. of Printed Pages - 3

Regd. No. | | | | | | | | | | |

Part – III
MATHEMATICS, Paper – I(A)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** Sections – **A**, **B** and **C**.

SECTION – A

10 × 2 = 20

I. Very Short Answer Type questions :

(i) Answer **all** the questions.

(ii) Each question carries **two** marks.

1. Find the domain of the real valued function $f(x) = \sqrt{x^2 - 25}$.

2. If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 3x - 1$, $g(x) = x^2 + 1$ then find (fog) (2).

3. Define a symmetric matrix. Give one example of order 3×3.

4. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$.

5. If the vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$ and $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear then find λ and μ.

6. Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).

7. Find the angle between the vectors $\bar{i} + 2\bar{j} + 3\bar{k}$ and $3\bar{i} - \bar{j} + 2\bar{k}$.

8. Find the value of $\sin 330° \cos 120° + \cos 210° \sin 300°$.

9. Find the extreme values of $\cos 2x + \cos^2 x$.

10. For any $x \in R$ show that $\cosh 2x = 2\cosh^2 x - 1$.

SECTION – B

$5 \times 4 = 20$

II. Short Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **four** marks.

11. If $A = \begin{bmatrix} 7 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 4 & 2 \\ -1 & 0 \end{bmatrix}$ then find AB' and BA'.

12. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors, prove that the following four points are co-planar $-\bar{a} + 4\bar{b} - 3\bar{c}$, $3\bar{a} + 2\bar{b} - 5\bar{c}$, $-3\bar{a} + 8\bar{b} - 5\bar{c}$ and $-3\bar{a} + 2\bar{b} + \bar{c}$.

13. Let $\bar{a}$ and $\bar{b}$ be vectors, satisfying $|\bar{a}| = |\bar{b}| = 5$ and $(\bar{a}, \bar{b}) = 45°$. Find the area of the triangle having $\bar{a} - 2\bar{b}$ and $3\bar{a} + 2\bar{b}$ as two of its sides.

14. If A is not an integral multiple of $\frac{\pi}{2}$ then prove that

    (i) $\tan A + \cot A = 2 \operatorname{cosec} 2A$ and

    (ii) $\cot A - \tan A = 2 \cot 2A$

15. Solve the equation $\sqrt{3} \sin \theta - \cos \theta = \sqrt{2}$.

16. Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$

17. If $a = (b - c) \sec \theta$, prove that $\tan \theta = \frac{2\sqrt{bc}}{b - c} \sin \frac{A}{2}$

SECTION – C $5 \times 7 = 35$

III. Long Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **seven** marks.

18. Let $f : A \to B$, $g : B \to C$ be bijections then prove that $gof : A \to C$ is a bijection.

19. Using mathematical induction, prove that :

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \text{ for all } n \in N.$$

20. Show that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\,(a+b+c)^3.$$

21. Solve the following system of equations by Gauss-Jordan Method :

$2x - y + 3z = 9$, $x + y + z = 6$ and $x - y + z = 2$.

22. For any four vectors $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$, prove that

(i) $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\ \bar{c}\ \bar{d}]\ \bar{b} - [\bar{b}\ \bar{c}\ \bar{d}]\ \bar{a}$ and

(ii) $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\ \bar{b}\ \bar{d}]\ \bar{c} - [\bar{a}\ \bar{b}\ \bar{c}]\ \bar{d}$

23. If A, B, C are the angles in a triangle, then prove that

$$\cos A + \cos B + \cos C = 1 + 4 \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right)$$

24. Show that in any triangle ABC

$r + r_3 + r_1 - r_2 = 4R \cos B.$