193

(I)

Total No. of Questions—24

Total No. of Printed Pages—4   **Regd. No.** | | | | | | | | | | |

Part III

MATHEMATICS

Paper I-B

(English Version)

Time : 3 Hours   ***Max. Marks : 75***

Note **:—** This question paper consists of THREE Sections A, B and C.

SECTION A   10×2=20

(I) Very short answer type questions :

(*i*) Attempt ALL questions.

(*ii*) Each question carries TWO marks.

1. Find the equation of the straight line passing through (–4, 5) and cutting off equal and non-zero intercepts on the co-ordinate axes.

2. If the area of the triangle formed by the straight lines $x = 0$, $y = 0$ and $3x + 4y = a$ $(a > 0)$ is 6, find the value of a.

3. Show that the points (1, 2, 3) (2, 3, 1) and (3, 1, 2) form an equilateral triangle.

4. Find the equation of the plane passing through (1, 1, 1) and parallel to the plane $x + 2y + 3z - 7 = 0$.

5. Compute :

$$\lim_{x \to 0} \frac{\sin ax}{\sin bx} \quad b \neq 0, a \neq b.$$

6. Compute :

$$\lim_{x \to \infty} \frac{11x^3 - 3x + 4}{13x^3 - 5x^2 - 7}.$$

7. Find the derivative of $e^{\sin^{-1} x}$.

8. Find the derivative of $\tan^{-1} \sqrt{\frac{1 - \cos x}{1 + \cos x}}$.

9. Find the approximate value of $\sqrt[3]{65}$.

10. Verify Rolle's theorem for $\sin x - \sin 2x$ on $[0, \pi]$.

SECTION B

5×4=20

(II) Short answer type questions :

(*i*) Attempt ANY FIVE questions.

(*ii*) Each question carries FOUR marks.

11. Find the equation of the locus of a point the difference of whose distance from (–5, 0) and (5, 0) is 8.

12. When the origin is shifted to (–1, 2) by the translation of axes find the transformed equation of $x^2 + y^2 + 2x - 4y + 1 = 0$.

13. Find the equation of the straight lines passing through (1, 3) and (*i*) parallel to (*ii*) perpendicular to the line passing through the points (3, –5) and (–6, 1).

14. If f, given by :

$$f(x) = \begin{cases} k^2x - k, & \text{if } x \geq 1 \\ 2, & \text{if } x < 1 \end{cases}$$

is a continuous function on R, then find the values of k.

15. Find the derivative of $\cos ax$ using first principle.

16. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of the edge is 10 centimeters ?

17. Find the lengths of subtangent subnormal at a point 't' on the curve $x = a(\cos t + t \sin t)$ $y = a(\sin t - t \cos t)$.

SECTION C

5×7=35

(III) Long answer type questions :

(*i*) Attempt ANY FIVE questions.

(*ii*) Each question carries SEVEN marks.

18. Find the ortho center of the triangle with the following vertices :

$$(-2, -1), (6, -1) \text{ and } (2, 5).$$

19. Show that the equation :

$$2x^2 - 13xy - 7y^2 + x + 23y - 6 = 0$$

represents a pair of straight lines, also find the angle between them and the co-ordinates of the point of intersection of the lines.

20. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

21. Show that the lines whose d.c.'s are given by $l + m + n = 0$, $2mn + 3nl - 5lm = 0$ are perpendicular to each other.

22. If

$$y = \tan^{-1}\left[\frac{2x}{1 - x^2}\right] + \tan^{-1}\left[\frac{3x - x^3}{1 - 3x^2}\right] - \tan^{-1}\left[\frac{4x - 4x^3}{1 - 6x^2 + x^4}\right]$$

then

$$\frac{dy}{dx} = \frac{1}{1 + x^2}.$$

23. Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

24. Find two positive numbers whose sum is 15 so that the sum of their squares is minimum.